Meriam 6/8: Linear Momentum Methods for 2D solid bodies

Noting that a solid body is just many particles glued together,

→ $\vec{G} = m\vec{V}_G$ = linear momentum

$$\vec{G}_2 = \vec{G}_1 + \int_{t_1}^{t_2} \Sigma \vec{F}\, dt$$

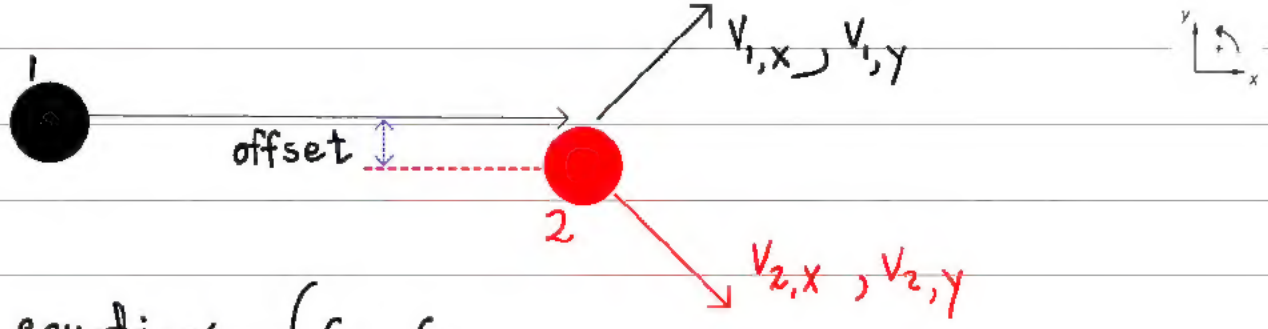
where

$\Sigma \vec{F}$ is the sum of all external forces

This equation is particularly useful for analyzing collisions, or explosions where unknown amounts of kinetic energy may be converted to heat, or generated from chemical reactions.

Even for elastic collisions, the energy equation can indicate the total kinetic energy following a collision, but not its distribution.

eg. billiard ball collision (elastic)



Need 4 equations to find 4 unknowns { G_x, G_y, T, offset

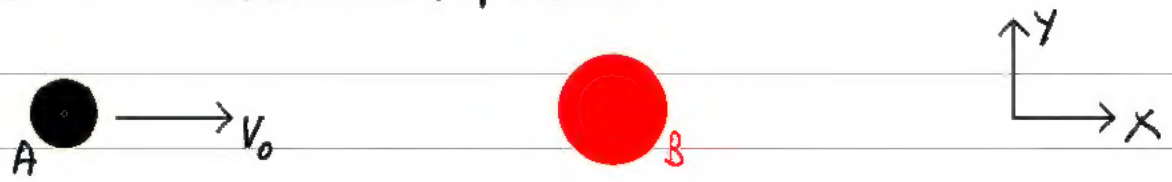
Eg.

Sphere A of mass m_A and velocity $V_0\hat{i}$ hits the center of a stationary sphere B of mass $2m_A$ on a horizontal surface.

(i) What are the velocities of A and B after the collision if it is elastic?

Answer:

Assume a 1-dimensional problem.



$$(G_1)_x = (G_2)_x$$

$$m_A V_0 = m_A V_{A2} + (2m_A) V_{B2}$$

$$T_1 = T_2$$

$$\tfrac{1}{2} m_A V_0^2 = \tfrac{1}{2} m_A V_{A2}^2 + \tfrac{1}{2} (2m_A) V_{B2}^2$$

$$\rightarrow V_0^2 = V_{A2}^2 + 4V_{B2}^2 + 4V_{A2}V_{B2} \quad (1)$$

$$V_0^2 = V_{A2}^2 + 2V_{B2}^2 \quad (2)$$

Subtract Eqn (2) from Eqn (1) $\rightarrow$ $\vec{V}_{A2} = -\frac{V_0}{3}\hat{i}$; $\vec{V}_{B2} = \frac{2V_0}{3}\hat{i}$

(ii) What fraction of energy is lost if A sticks to B?

Answer:

$$(G_1)_x = (G_2)_x$$

$$m_A V_0 = (m_A + \underbrace{m_B}_{2m_A}) V_2$$

$$\rightarrow V_2 = V_0/3$$

$$\text{fraction of energy lost} = 1 - \frac{(\tfrac{1}{2})(3m_A)(\frac{V_0}{3})^2}{\tfrac{1}{2} m_A V_0^2} = 1 - \tfrac{1}{3} = \boxed{\tfrac{2}{3}}$$

Eg.

Given:

$m_{AB} = 3\,kg$

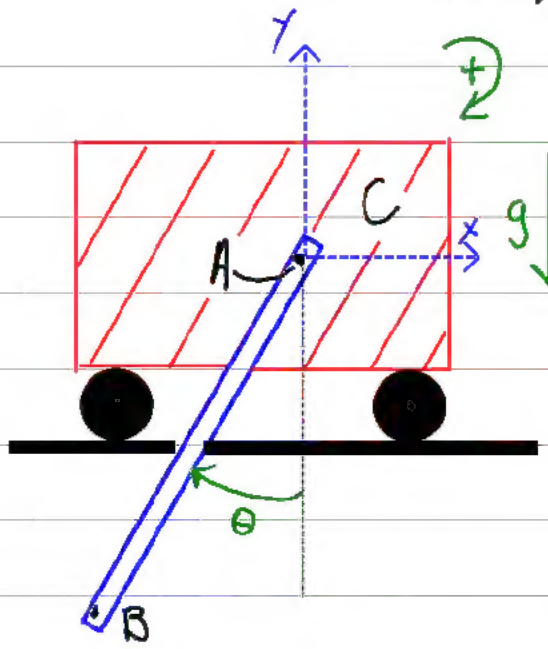
$L_{AB} = 1\,m$

$m_C = 5\,kg$

$V_0 = \omega_0 = 0$

$\theta_0 = 30°$

Zero friction



System is released from rest at $t = 0$.

Find v_c when $\theta = 0$

Solution

try: $\alpha_{AB}(t) = \dfrac{\sum \vec{M}_G}{I_G}$

& $\vec{\omega}_{AB} = \vec{\omega}_0 + \int \vec{\alpha}_{AB}(t)\,dt$

& $\vec{V}_B = \vec{V}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$

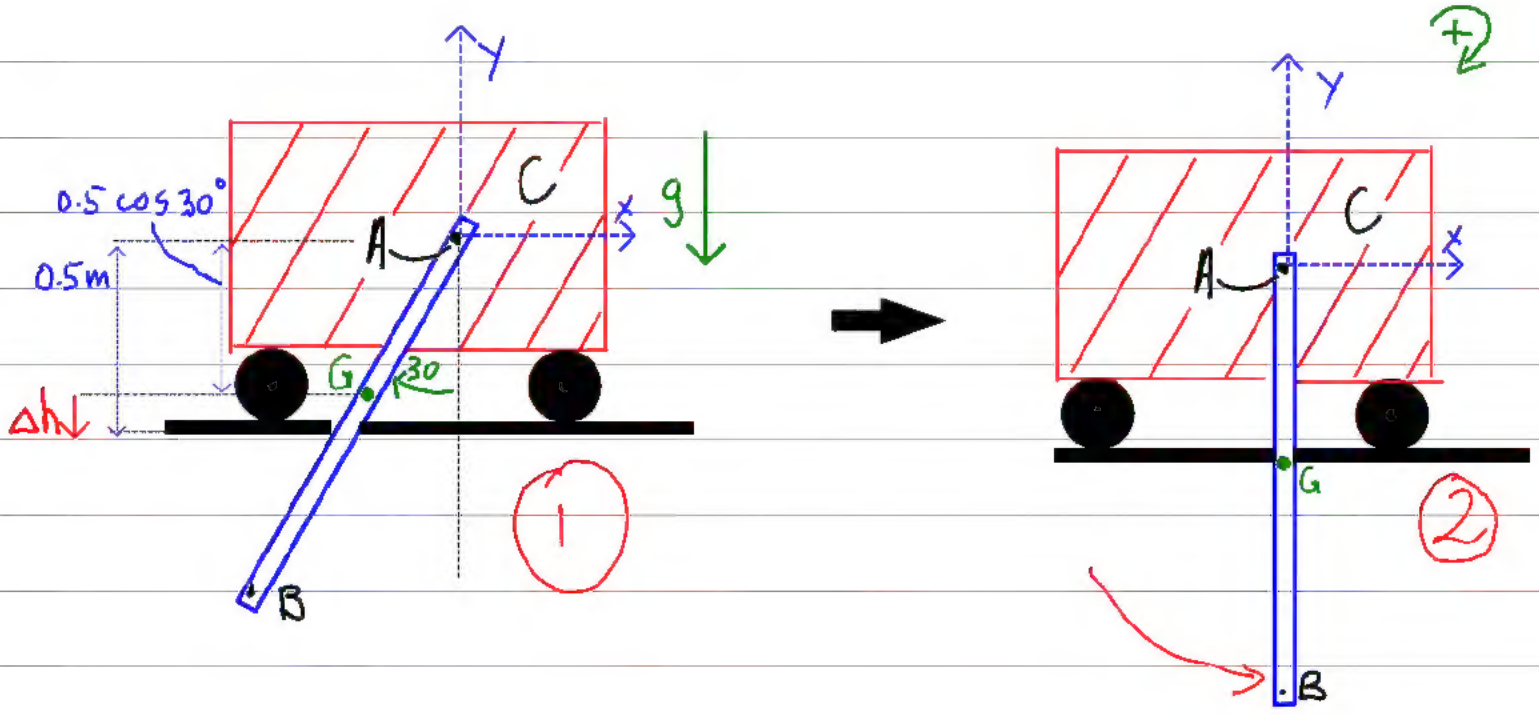
Note we are given initial configuration & velocities, and asked to find $\vec{V}$, ω after a trip.

→ energy methods / momentum methods

Define position ① as initial position, with $\theta = 30°$
Define position ② as configuration where $\theta = 0°$

For entire system of cart C plus bar AB,

(a) $T_2 = \cancel{T_1}^{\,0} + U_{1\to 2}$



$$\tfrac{1}{2} m_C (V_C)_2^2 + \tfrac{1}{2} m_{AB} (V_G)_2^2 + \tfrac{1}{2} I_{AB} (\omega_{AB})_2^2 = - m_{AB} g \Delta h_G$$

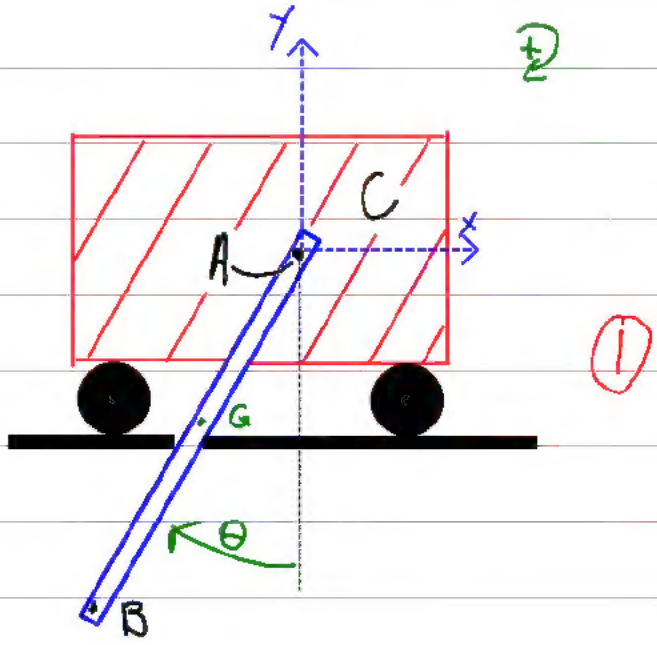
$$\rightarrow (\tfrac{1}{2})(5\,kg)(V_C)_2^2 + (\tfrac{1}{2})(3\,kg)(V_G)_2^2 + (\tfrac{1}{2})\left[(\tfrac{1}{12})(3\,g)(1m)^2\right](\omega_{AB})_2^2 =$$

$$= - (3kg)(9.81)\left[-0.5m(1-\cos 30°)\right]$$

→ 1 equation: 3 unkowns.

3 unknowns :

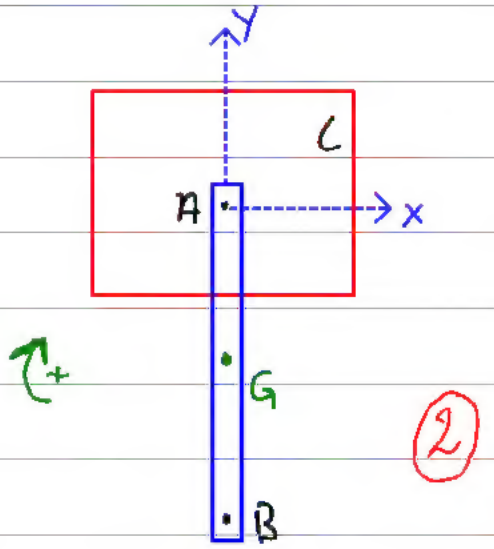
$(V_C)_2$
$(V_G)_2$
$(\omega_{AB})_2$



(b) when $\theta = 0$ at final position ②,

$$(\vec{V}_G)_2 = (\vec{V}_A)_2 + (\vec{V}_{G/A})_2$$

A, C, B, G are all moving along +/− x-axis



$$\rightarrow (V_G)_2 = (V_C)_2 - (\omega_{AB})_2 r_{AG}$$

(c) $$\vec{G}_2 = \vec{G}_1 + \int_{t_1}^{t_2} \vec{F}\, dt$$

In x-direction, $(5\text{ kg})(V_C)_2 + (3\text{ kg})(V_G)_2 = 0$

$$\rightarrow \quad \omega = -2.342\ s^{-1}$$
$$(\vec{V}_C)_2 = -0.439\ m/s$$
$$(\vec{V}_G)_2 = 0.732\ m/s$$

} or the opposite? (quadratic equation)

Summary

$$\vec{G}_2 = \vec{G}_1 + \int_{t_1}^{t_2} \Sigma \vec{F}\, dt$$

- Particularly useful for collisions/explosions
  → Equation can be applied over a very short time period
- Momentum equation is valid even if energy is lost due to heat generation.

Next Lecture:

- Meriam Chapter 6/8: Angular Momentum
- Angular momentum problems